ON FSGB-CONTINUOUS, IRRESOLUTE, OPEN AND CLOSED MAPPINGS IN FUZZY TOPPOLOGICAL SPACES.

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ABSTRACT
A new form of fuzzy strongly generalized b-continuous, fuzzy strongly generalized b-irresolute, fsgb-open, and fsgb-closed mappings are introduced in fuzzy topological spaces. Also, some of its characteristics and properties are examined.
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1. Introduction:

Mapping is a crucial technique for understanding topological concepts and creating new topological spaces out of pre-existing ones. The broad idea of mapping is overly focused on topology and analysis. Imagine a topological space as a set from which all structures that are not essential to the continuity of mappings defined on it have been swept away. The notions of continuous, open, and closed mappings in fuzzy topological spaces were expanded by Chang (1968)[`7]`. Fuzzy semi continuity was first described by K. K. Azad [3] in 1981. Generalized fuzzy continuous functions were introduced by G. Balasubramaniam and P. Sundaram [2] in 1997, while fuzzy semi-pre continuity was introduced by S. S. Thakur and S. Singh [9] in 1998. And fb-continuity, fgb and fbg-continuity were introduced by S. S. Benchalli and Jenifer Karnel [4,5,6] in 2010.Andrijevic was the first to develop the concept of b-open sets in general topology [1]. Jenifer and Megha introduced the concept of fuzzy strongly generalized b-open and closed sets in [8].

This article introduces the concepts of fsgb-closure ,fsgb-interior and fsgb-fsgb-continuous, fsgb- irresolute mapping, fsgb-open, and fsgb-closed maps in FTS. Their properties are determined as well as some characterizations.

2. Preliminaries:

Throughout this study(L,τ),(M,σ) and,(N,γ) (or simply L,M,and N) are fuzzy topological spaces (in short, fts). The interior, closure, and compliment of a fuzzy subset P of (L,τ) are denoted by Int(P) , Cl(P), and P^crespectively. Unless otherwise specified, no separation axioms are expected.

2.1 Definition[4] A fuzzy set$P$ in a fts L is called fb-open iff $P \leq (\mathrm{IntCl}(P) \vee \mathrm{ClInt}(P))$.

2.2 Definition[4]Fb-interior and Fb-closure of afuzzy set $P$is as follows

(i)bInt(P) =$\vee\{Q: Q$ is a fb-open set of L and $P \geq Q\}$.

(ii) bCl(P) =$\wedge\{R: R$ is a fb-closed set of L and $R \geq P\}$.

2.3Defintion [2] A fuzzy set(f-set) P in a FTS L is known as fuzzy generalized closed set(in short(fg-CS) if $\mathrm{Cl}(P) \leq Q$, whenever $P \leq Q$ and Q is fuzzy open set(f-OS) in L.

2.4Definition [8]A f-OS P in a fts L is called a fsgb-CS if bCl$(P) \leq Q$, whenever $P \leq Q$and $Q$is fg -open set in$L$.

2.5Definition [8]A f-OS P in a fts L is called a fsgb-open set if bInt$(P) \geq Q$, whenever $P \geq Q$and Q is fg -open set in L.

2.6Definition: A fuzzy set P in a FTS (L,τ) is known as
i)fuzzy generalized b-closed set[3](in short fgb-CS) if bCl$(P) \leq Q$ where $P \leq Q$ and Q is f-OS in L.
ii)fuzzy semi open set[3](in short fs-OS) if $P \leq$ Cl(Int$P$).
iii)fuzzy α-open set[3](in short fα-OS) if $P \leq$ IntCl(Int$P$).
iv)fuzzy pre-open set[3](in short fp-OS) if $P \leq$ Int(ClP).
2.7 Definition: Let M, N be two fts. A mapping $g: L \to M$ is called
i)fuzzy continuous map(in short f- ℂℕ map)[7] if $g^{-1}(P)$ is f-OS in L ,for every f-OS P of M.
ii)fuzzy generalized continuous map(in short fg- ℂℕ map)[2] if$g^{-1}(Q)$is fg-closed in L, for every f-CS Q of M.
iii)fuzzy b-continuous map(in short fb-ℂℕ map) [5] if $g^{-1}(P)$ is fb-CS in L, for every f-CS P of M.
iv)fuzzy b*-continuous map(in short fb*- ℂℕ map) [5] if$g^{-1}(P)$ is fb-CS in L , for every fb-CS P of M.
v)fuzzy b-closed map(fb-CM)[5] if $g(P)$ is fb-CS in M , for every f-CS P in L.
vi)fuzzy b*-closed map(fb*-CM)[5] if $g(P)$ is fb-CS in M , for every fb-CS P in L.

3. Fuzzy Strongly Generalized b-closure and interior.

The definitions of fsgb-interior(fsgb- Int) and fsgb-closure (fsgb- Cl) of a f-set are defined and their characteristics are established in this section.

Definition 3.1. If P is a f-set in a fts, then fsgb $- Cl(P) = \wedge \{Q: Q$ is a fsgb-CS and $P \leq Q\}$ fsgb-$Int(P) = \vee \{Q: Q$ is a fsgb-OS and $P \geq Q\}$.

Theorem3.2. If P be a fsgb-CS in a fts (Y, τ), then $P =$ fsgb-$Cl(P)$.

Proof: Consider P be a fsgb-CS in a fts (Y, τ). Then we have fsgb-$Cl(P) \leq P$. But $P \leq$ fsgb-$Cl(P)$ always, thus $P =$ fsgb-$Cl(P)$.

Theorem3.3. The following results hold for fsgb-Cl in fts Y.
(i) fsgb-$Cl(0) = 0$,fsgb-$Cl(1) = 1$.
(ii) $P \leq$ fsgb-$Cl(P) \leq$ f-$Cl(P)$.
(iii) fsgb-$Cl($fsgb-$Cl(P)) =$ fsgb-$Cl(P)$.

Proof. Consider P be a f-set in a fts (Y, τ).
(i) Obvious.
(ii) Every f-CS is an fsgb-CS by property fsgb-$Cl(P) \leq$ f-$Cl(P)$ and by definition, $P \leq$ fsgb-$Cl(P)$, Thus $P \leq$ fsgb-$Cl(P) \leq$ f-$Cl(P)$.
(iii) Since fsgb-$Cl(P)$ is fsgb-closed.

Remark3.4. For any 2 f-sets P and Q, fsgb-$Cl(P) =$ fsgb-$Cl(Q)$ does not imply that $P = Q$. The following example illustrates this.

Example3.5. Consider $Y = \{d, e, f\}$
$R = \{(d, 1), (e, 0), (f, 0)\}$
$S = \{(d, 1), (e, 0), (f, 1)\}$
$P = \{(d, 0), (e, 0), (f, 1)\}$
$Q = \{(d, 0), (e, 1), (f, 1)\}$ be f-sets of Y.

Consider $\tau = \{0, R, S, 1\}$, Then fsgb-$Cl(P) =$ fsgb-$Cl(Q) = 1$.

It implies that fsgb-$Cl(P) =$ fsgb-$Cl(Q)$ but $P \neq Q$.

Theorem3.6. The following results hold for fsgb-closure in a fts Y.
(i) fsgb-$Cl(P) \leq$ fsgb-$Cl(Q)$ if $P \leq Q$.

(ii) fsgb-$Cl(P)$ ∨ fsgb-$Cl(Q) \leq$ fsgb-$Cl(P \vee Q)$.
(iii) fsgb-$Cl(P \wedge Q) \leq$ fsgb-$Cl(P)$ ∧ fsgb-$Cl(Q)$.
Proof . Consider P and Q be a f-sets in fts (Y, τ)
(i) Since $P \leq Q)$, a fsgb-CS containing Q, contains P also. Thus fsgb-$Cl(P) \leq$ fsgb-$Cl(Q)$.
(ii) Since $P \leq Q)$, a fsgb-CS containing Q, contains P also. Thus fsgb-$Cl(P) \leq$ fsgb-$Cl(Q)$.
(iii) Consider $P \wedge Q \leq P$ and $P \wedge Q \leq Q$. This implies fsgb-$Cl(P \wedge Q) \leq$ fsgb-$Cl(P)$ and fsgb-$Cl(P \wedge Q) \leq$ fsgb-$Cl(Q)$ by (i) Thus fsgb-$Cl(P \wedge Q) \leq$ fsgb-$Cl(P)$ ∧ fsgb-$Cl(Q)$.
Theorem 3.7: The following results hold for fsgb-Interior in fts Y.
(i) fsgb-$Int(0) = 0$,fsgb-$Int(1) = 1$.
(ii) f-$Int(P) \leq$ fsgb-$Int(P) \leq (P)$.
(iii) fsgb-$Int\big($fsgb-$Int(P)\big) =$ fsgb-$Int(P)$.
Proof. Consider P be a f-set in a fts (Y, τ).
(i) Obvious.
(ii) Every f-OS is a fsgb-OS By theorem f-$Int(P) \leq$ fsgb-$Int(P)$ and bydefinitionfsgb-$Int(P) \leq P$, Thus f-$Int(P) \leq$ fsgb-$Int(P) \leq P$.
(iii) By theorem fsgb-$Int\big($fsgb-$Int(P)\big) =$ fsgb-$Int(P)$, as fsgb-$Int(P)$fsgb-OS.
Remark 3.8. For any 2 f-sets P and Q, fsgb-$Int(P) =$ fsgb-$Int(Q)$ does not imply that $P = Q$. The following example illustrates this.
Example3.9. Consider $Y = \{d, e, f\}$
$R = \{(d, 1), (e, 0), (f, 0)\}$
$S = \{(d, 1), (e, 0), (f, 1)\}$
$P = \{(d, 0), (e, 0), (f, 1)\}$
$Q = \{(d, 0), (e, 1), (f, 1)\}$Be f-sets of Y.

Consider $\tau = \{0, R, S, 1\}$, then fsgb-$Int(P) =$ fsgb-$Int(Q) = 1$.

It implies that fsgb-$Int(P) =$ fsgb-$Int(Q)$ but $P \neq Q$.

Theorem3.10. The following results hold for fsgb-Interior in a fts Y.

(i) fsgb-$Int(P) \leq$ fsgb-$Int(Q)$ if $P \leq Q$.

(ii) fsgb-$Int(P)$ ∨ fsgb-$Int(Q) \leq$ fsgb-$Int(P \vee Q)$.

(iii) fsgb-$Int(P \wedge Q) \leq$ fsgb-$Int(P)$ ∧ fsgb-$Int(Q)$.
Proof. Consider P and Q be a f-sets in fts (Y, τ)
(i) Since $P \leq Q$, a fsgb-CS containing Q, contains P also. Thus fsgb-$Int(P) \leq$ fsgb-$Int(Q)$.
(ii) Consider $P \leq P \vee Q$ and $Q \leq P \vee Q$. Thus, implies fsgb-$Int(P) \leq$ fsgb-$Int(P \vee Q)$ and fsgb-$Int(Q) \leq$ fsgb-$Int(P \wedge Q)$ by (i). Thus fsgb-$Int(P)$ ∨ fsgb-$Int(Q) \leq$ fsgb-$Int(P \vee Q)$.
(iii) Consider $P \wedge Q \leq P$ and $P \wedge Q \leq Q$. This implies fsgb-$Int(P \wedge Q) \leq$ fsgb-$Int(P)$ and fsgb-$Int(P \wedge Q) \leq$ fsgb-$Int(Q)$ by (i) Thus fsgb-$Int(P \wedge Q) \leq$ fsgb-$Int(P)$ ∧ fsgb-$Int(Q)$.

4. Fsgb-neighbourhood and fsgb-q-neighbourhood.

fsgb-neighbourhood and fsgb-q-neighbourhood of a fuzzy point is introduced along with its characteristics.
Definition4.1. Consider Q be a f-set in fts Y and y_P be a fuzzy-point in Y, the Q is called fsgb-neighborhood (briefly fsgb-n) of y_P iff there exists a fsgb-OS R such that $y_P \in R \leq Q$.
Theorem4.2. A f-set Q is a fsgb-OS in Y iff for every fuzzy-point $y_P \in Q$, Q is a fsgb-n of y_P.
Proof. Consider Q be fsgb-OS in Y.For every $y_P \in Q$, consider $Q \leq Q$. Hence Q is a fsgb-n of y_P.

Conversely, consider Q be a fsgb-n of y_P then by definition, there exists a fsgb-OS R such that $y_P \in R \leq Q$. Thus Q is fsgb-OS in Y.

Theorem4.3. If Q an R are fsgb-n of y_P then $Q \wedge R$ is also a fsgb-n of y_P.

Proof. Consider Q and R be fsgb-n of y_P. Thus, there exist fsgb-OS M and N such that $y_P \in M \wedge N \leq Q \wedge R$. Therefore $Q \wedge R$ is also a fsgb-n of y_P.

Definition4.4. In a fts L, a f-set P is said to be fuzzy strongly generalized b-q-neighborhood (in short, fsgb-q-n) of a fuzzy point l iff there exists a fsgb-OSQ in L such that $l\, q\, Q \leq P$.

Theorem4.5. If $P$and Q be fsgb-n of l then there exists fsgb-OSR and S such that $l \in R \leq P$ and $l \in S \leq Q$ and $l \in R \wedge S \leq P \wedge Q$. Thus, $P \wedge Q$ is also fsgb-n of l.

Theorem4.6. A f-set P in a fts L is fsgb-OS in L iff, for every fuzzy point $l \in P$, P is fsgb-n of l.

Proof. Consider P be fsgb-OS in L and fuzzy point $l \in P$. Since $P \leq P$, thus $l \in P \leq P$. Therefore, P is fsgb-n of l.

Conversely, consider P be fsgb-n of l. There exists a fsgb-OSR in L, such that $l \in R \leq P$. Thus, P is fsgb-OS in L.

Theorem4.7. Let P be f-set in L. Then, a fuzzy point $l \in$ fsgb $Cl(P)$ iff each fsgb-q-n of l is quasi-coincident with P.

Proof. Let $l \in$ fsgb $Cl(P)$. Let Q be fsgb-q-n of l such that $Q_q P$. There exist fsgb-OSR in L such that $l\, q\, R \leq Q$. This implies $R\, q\, P$. Thus, $P \leq 1 - R$, $1 - R$ is fsgb-CS in L and fsgb $Cl(P) \leq 1 - P$. Since $l \in 1 - R$ implies, $l \notin$ fsgb $Cl(P)$ which is a contradiction.

Conversely, since every fsgb-q-n of l is quasi-coincident with P. Let $l \notin$ fsgb $Cl(P)$. Then there exists a fsgb-CSQ, such that $P \leq Q$ and $l \notin Q$. Therefore, $1 - Q$ is fsgb-OS such that $l\, q(1 - Q)$ and $(1 - Q)\, q\, P$ which is not true.

Theorem4.8. Let P be a f-set in fts L. Then, $P$is fsgb-CSiff$P\, \overline{q} Q \Rightarrow$ fsgb $Cl(P)\overline{q}Q$, for each fb-CS in L.

Proof. Consider P be fsgb-CS and Q be fb-CS so that $P\, \overline{q} Q$. Then, from the definition of quasi-coincident $P \leq 1 - Q$ where $1 - Q$ is fb-OS in L. Since P is fsgb-CS, fsgb $CL(P) \leq 1 - Q$. Thus,fsgb $Cl(P)\overline{q}\, Q$.

Conversely, consider D be fb-OS in L and P be f-set in L, such that $P \leq D$. Since $P\overline{q}(1 - D)$ where $1 - D$ is fb-CS in L, this implies that fsgb $Cl(P)\overline{q}(1 - D)$, from the definition fsgb $Cl(P) \leq D$ obtained. Therefore, P is fsgb-CS in L.

Theorem4.9. For fsgb-CSP in L and fuzzy point l of L, such that $l\, q$ fsgb $Cl(P)$, then fsgb $Cl(l)qP$.

Proof. Let P be fsgb-CS and l be fuzzy point of L. If fsgb $Cl(l)\overline{q}P$, then by definition fsgb $Cl(l) \leq 1 - P$, this implies $P \leq 1$ fsgb $Cl(l)$. Thus, fsgb $Cl(P) \leq 1 -$ fsgb $Cl(l) \leq 1 - l$. As $1 -$ fsgb $Cl(l)$ is fb-OS in L and P is fsgb-CS in L. Thus, $l\, \overline{q}$fsgb $Cl(P)$ which is a contradiction.

5.Fsgb-Continuous maps in fts.

Definition5.1. A map $g: L \rightarrow M$ is known as fsgb-ℂℕ map that is fsgb-continuous map if $g^{-1}(P)$ is fsgb-closed set in L, for every f-CS P in M.

Theorem 5.2. A function $g: L \rightarrow M$is fsgb-ℂℕ map iff the inverse image of every fg-OS of M is fsgb-OS of L.

Proof. Let Q be a fsgb-OS of M. $1 - Q$ is fsgb-CS in M. Since $g: L \rightarrow M$ is fsgb-ℂℕ map,$g^{-1}(1 - Q) = 1 - g^{-1}(Q)$ is fsgb-closed set of L .Hence $g^{-1}(Q)$ is fsgb-open set of L. The converse is obvious.

Definition 5.3. A map $g: L \rightarrow M$is known as fsgb-ℂℕ map if the inverse-image of each fg-OS in M is fsgb-OS in L.

Theorem 5.4. Every f- ℂℕ map is fsgb- ℂℕ map.

Proof: Let $\hbar: L \rightarrow M$ be a f-ℂℕ map. Let P be f-OS in M. Since $\hbar$ is fuzzy- ℂℕ map, $\hbar^{-1}(P)$is f-OS in L. Also $\hbar^{-1}(P)$ is fsgb-OS in L. Hence $\hbar$ is fsgb- ℂℕ map.

The below example shows the opposite of this theorem is incorrect.

Example 5.5. Let $L = M = \{e, f\}$ and let $= \{(e, 1), (f, 0.9)\}$, $Q = \{(e, 0.5), (f, 0.4)\}$
Consider $\tau = \{0,1, P\}$ and $\sigma = \{0,1, Q\}$
$bO(L) = \{0,1, P, (e, \lambda), (f, \kappa)\}$ where $\lambda > 0$ or $\kappa > 0.1$
$bC(L) = \{0,1, P, (e, \lambda), (f, \kappa)\}$ where $\lambda = 0$ or $\kappa < 0.1$
Then (L, τ) and (M, σ) are fts. Let $g: L \to M$ be an identity map. Then g is fsgb-ℂℕ map but not f-ℂℕ map, as f-OS Q in M, $g^{-1}(Q)$is not f-CS in L but it is fsgb-CS in L.

Theorem 5.6. Every f-ℂℕ(fs-ℂℕ , fα- ℂℕ, fp- ℂℕ , fb- ℂℕ , fgb- ℂℕ,fbg- ℂℕ,) map is fsgb-ℂℕ map.

Proof. Let $g: L \to M$ is f-ℂℕ(fs-ℂℕ , fα- ℂℕ, fp- ℂℕ , fb- ℂℕ , fgb- ℂℕ, fbg- ℂℕ) map. Let P be f-CS in Since g is f-ℂℕ(fs-ℂℕ , fα- ℂℕ, fp- ℂℕ , fb- ℂℕ , fgb- ℂℕ, fbg- ℂℕ) map, $g^{-1}(P)$ is a f-CS (fs-CS , fα-CS, fp-CS, fb-CS, fgb-CS, fbg- ℂℕ) in L . And so $g^{-1}(P)$ is a fsgb-CS in L. Therefore $g$is fsgb-ℂℕ map.

The below illustration shows that the inverse implication of the theorem 5.6 is not true.

Example 5.7. Let $L = \{d, e\}$ and $M = \{i, j\}$.
Then $P = \{(d, 0.3), (e, 0.4)\}, Q = \{(d, 0.7), (e, 0.5)\}, R = \{(d, 0.7), (e, 0.7)\}$.
Let $= \{0,1, P\}$, $\sigma = \{0,1, Q\}$
Then function $g: L \to M$ defined by $g(d) = i$ and$g(e) = j$.
Then the fuzzy set R is a f-CS in M and $g^{-1}(R)$ is not a f-CS (fs-CS, fα-CS, fp-CS, fb-CS, fgb-CS) in L , but a fsgb-CS in L. Hence g is a fsgb- ℂℕ map but not fuzzy- ℂℕ map.

Theorem 5.8. Consider $g: (L, \tau) \to (M, \sigma)$ be fsgb-ℂℕ. Then [fsgb-$Cl(P)$] where P is any f-set in L.

Proof. Consider P be any f-set in L. So that $Cl[g(P)]$ is a f-CS in L. Since g is fsgb-ℂℕ, $g^{-1}\big(Cl(g(P))\big)$ and so fsgb-$Cl(P) \leq g^{-1}\big(Cl(g(P))\big)$.
Therefore ,g[fsgb-$Cl(P)$] $\leq Cl[g(P)]$.

Theorem 5.9. Consider $g: (L, \tau) \to (M, \sigma)$ be a fsgb-ℂℕ and L is fsgb$T_{\frac{1}{2}}$ space, then g is a f-ℂℕ map.

Proof. Consider $g: (L, \tau) \to (M, \sigma)$ is fsgb-ℂℕ. Let P be f-CS in M. Then $g^{-1}(P)$ is a fsgb-CS in L, since g is fsgb-ℂℕ. As L is fsgb$T_{\frac{1}{2}}$ space, $g^{-1}(P)$ is a f-CS in M. Thus, g is f-ℂℕ.

Theorem 5.10. Consider $g: L \to M$ be a fsgb-ℂℕ, $h: M \to N$ is fsgb-ℂℕ and M is fsgb$T_{\frac{1}{2}}$ space, then $h \cdot g: L \to N$ is a fsgb-ℂℕ map.

Proof. Consider Q be a f-CS in N, then $h^{-1}(Q)$ is a fsgb-CS in M, as h is fsgb-ℂℕ. Since M is fsgb$T_{\frac{1}{2}}$ space, $h^{-1}(Q)$ is a f-CS in M. And then $g^{-1}[h^{-1}(P)]$ is a fsgb-CS in L as g is fsgb-ℂℕ. Now $(h \cdot g)^{-1}(P) = g^{-1}[h^{-1}(P)]$ is a fsgb-CS in L. Thus, $h \cdot g: L \to N$ is fsgb-ℂℕ map.

Definition 5.11. A mapping $g: L \to M$ is known as fsgb-irr that is fsgb-irresolute map, if $g^{-1}(P)$ is fsgb-CS in L for every fsgb-CS P in M.

Theorem 5.12. A mapping $g: L \to M$ is fsgb-irr iff the inverse of every fsgb-OS in M is fsgb-OS in L.

Proof. It follows from definition 5.11.

Theorem 5.13. Every fsgb-irresolute mapping is fsgb-ℂℕ map.

Proof. Let $\hbar: L \to M$ is fsgb-irr. Let P be fuzzy closed in M, it follows that P is fsgb-CS in M. Since $\hbar$ is fsgb-irr then the inverse image of P is fsgb-CS in L. Therefore $\hbar$ is fsgb-ℂℕM.

The below example shows the opposite of this theorem is incorrect.

Example 5.14. Let $L = M = \{x, y\}$ and the fuzzy sets P, Q, R, S and T be defined as follows, $P = \{(x, 0.8), (y, 0.8)\}$, $Q = \{(x, 0.7), (y, 0.4)\}$, $R = \{(x, 0.6), (y, 0.4)\}$, $S = \{(x, 0.4), (y, 0.1)\}$and $T = \{(x, 0.4), (y, 0.5)\}$.
Let $\tau = \{0,1, P, Q, R, S\}$ and $\sigma = \{0,1, T\}$ then (L, τ) and(M, σ) are fts.

Define $g: L \to M$ by $g(x) = z$, $g(y) = x$ and $g(z) = y$.
Then g is fsgb-ℂℕ map but not fsgb-irr map, as the fuzzy set T is fsgb-CS in M
But $g^{-1}(T) = R$ is not fsgb-CS in L.

Theorem 5.15. Let $g: L \to M$and $h: M \to N$ be two mappings then
(i) $h \cdot g: L \to N$ is fsgb-ℂℕ map , if g is fsgb-ℂℕ map and h is fuzzy-ℂℕ map.
(ii) $h \cdot g: L \to N$ is fsgb-irr map , if g and $\hbar$ are fsgb-irr map.
(iii) $h \cdot g: L \to N$ is fsgb- ℂℕmap , if g is fsgb-irr map and $\hbar$ is fsgb-ℂℕ map.

Proof.
(i) Suppose that P be f-CS of N. As $h: M \to N$ is f-ℂℕ, so $g^{-1}(P)$ is f-CS of M. And $g: L \to M$ is fsgb-ℂℕ , $g^{-1}(P)$ is f-CS of M, thus by definition 3.3 $g^{-1}(h^{-1}(P)) = (h \cdot g)^{-1}(P)$ is fsgb-CS in L. Therefore $h \cdot g: L \to N$ is fsgb-ℂℕ map.
(ii) Consider $h: M \to N$ is fsgb-irr and let P be fsgb-CS of N. As h is fsgb-irr by definition 3.9 $h^{-1}(P)$ is fsgb-CS of M. And $g: L \to M$ is fsgb-irr, then $g^{-1}(h^{-1}(P)) = (h \cdot g)^{-1}(P)$ is fsgb-CS. Hence $h \cdot g: L \to N$ is fsgb-irr map.
(iii) Suppose that P be f-CS of N. As $h: M \to N$ is fsgb-ℂℕ , $h^{-1}(P)$ is fsgb-CS of M. And $g: L \to M$ is fsgb-irr, so every fsgb-CS of M is fsgb-CS in L. Thus $g^{-1}(h^{-1}(P)) = (h \cdot g)^{-1}(P)$ is fsgb-CS of L. Therefore $h \cdot g: L \to N$ is fsgb-ℂℕ map.

Theorem 5.16. If $g: L \to M$ be a fsgb-ℂℕ, $h: M \to N$ be fsgb-irr mapping and M is fsgb$T_{\frac{1}{2}}$ space, then $h \cdot g: L \to N$ is a fsgb-irr mapping.

Proof. Consider Q be fsgb-CS in N, then $h^{-1}(Q)$ is a fsgb-CS in M as h is fsgb-irr. Since M is fsgb$T_{\frac{1}{2}}$ space, $h^{-1}(Q)$ is a f-CS in M. Since g is fsgb-ℂℕ, $g^{-1}[h^{-1}(Q)]$ is a fsgb-CS in L. Now $(h \cdot g)^{-1}(Q) = g^{-1}[h^{-1}(Q)]$ is a fsgb-CS in L. Now $(h \cdot g)^{-1}(Q) = g^{-1}[h^{-1}(Q)]$ is a fsgb-CS in L. Thus $h \cdot g: L \to N$ is a fsgb-irr mapping.

Theorem 5.17. Consider L and M be fsgb$T_{\frac{1}{2}}$ space. Then $g: (L, \tau) \to (M, \sigma)$ the following are equivalent:
(i) g is a fgb-irr mapping.
(ii) g is a fsgb-irr mapping.

Proof:
(i)→(ii) Consider $g: (L, \tau) \to (M, \sigma)$ be fsgb-irr map. Let g be fsgb-CS in M. As M is fsgb$T_{\frac{1}{2}}$ space, then Q is a fgb-CS in M. Since g is fgb-irr, $g^{-1}(Q)$ is a fgb-CS in L. And every fgb-CS is fsgb-CS and hence, $g^{-1}(Q)$ is a fsgb-CS in L. Thus, g is a fsgb-irr mapping.
(ii)→(i) Consider $g: (L, \tau) \to (M, \sigma)$ be fsgb-irr map. Consider R be fgb-CS in M and so R is a fsgb-CS in M, as every fgb-CS is fsgb-CS. Since g is fsgb-irr, $g^{-1}(R)$ is a fsgb-CS in L. But L is fsgb$T_{\frac{1}{2}}$ space, $g^{-1}(R)$ is a fgb-CS in L. Thus, g is a fgb-irr mapping.

6. Fsgb-Open Map and Fsgb-Closed Maps in FTS.

Definition 6.1. A mapping $g: L \to M$ is known as fsgb-OM that is fsgb-open map if the image of every fuzzy-OS in L is fsgb-OS in M.

Definition 6.2. A mapping $g: L \to M$ is known as fsgb-CM that is fsgb-closed map if the image of every fuzzy-CS in L is fsgb-CS in M.

Definition 6.3. A mapping $g: L \to M$ is known as fsgb*-OM that is fsgb*-open map if the image of every fsgb-OS in L is fsgb-OS in M.

Definition 6.4. A mapping $g: L \to M$ is said to be fsgb*-CM that is fsgb*-closed map if the image of every fsgb-CS in L is fsgb-CS in M.

Example 6.5. Consider $= \{d, e, f\} M = \{d, e, f\}$.
Let the f-sets be $P = \{(d, 0.5), (e, 0.4), (f, 0.7)\}$,
$Q = \{(d, 0.8), (e, 1), (f, 0.4)\}$ and $R = \{(d, 0.5), (e, 0.5), (f, 0.3)\}$
Let $\tau = \{0, P, 1\}$ and $\sigma = \{0, Q, 1\}$

Define the mapping $g:(L,\tau)\to(M,\sigma)$ by $g(d)=g(e)=d$ and g(f) = f.Then the only f-CS in L is R and $g(R)$ is fsgb-CS in M. Thus, g is a fsgb-CM.

Remark 6.6. Every fsgb*-open (fsgb*-closed) map is fsgb-open (fsgb-closed) map.

The converse of this theorem is incorrect.

Example 6.7. Let $L=\{d,e\}$ and $M=\{i,j\}$.

Then $P=\{(d,0.2),(e,0.4)\},Q=\{(d,0.6),(e,0.7)\}$,

$R=\{(d,0.6),(e,0.7)\}$.

Let= $\{0,1,P\}$, $\sigma=\{0,1,Q\}$

Then function$g:L\to M$ defined by $g(d)=i$ and $g(e)=j$ is fsgb-OM ,but not fsgb*-OM.

Theorem 6.8. If $g:L\to M$ is fg-CM and $h:M\to N$is fsgb-CM then is $h\circ g$ is fsgb-CM.

Proof: For a fg-CS in L, $g(P)$is fg-CS in M. Since $h:M\to N$ is fsgb-CM, $h\big(g(P)\big)$ is fsgb-CS in *N*. $h\big(g(P)\big)=(h\circ g)(P)$ is fsgb-CS in *N*. Hence $h\circ g$is fsgb-CM.

Theorem 6.9. Let $g:L\to M,h:M\to N$ be two mappings then $h\circ g:L\to N$ is fsgb-CM.

(i) If g is f-ℂℕ and surjective then h is fsgb-CM.

(ii) If h is fsgb-irr and injective, then g is fsgb-CM.

Proof:

(i) Let Q be fuzzy-CS of M. Then $g^{-1}(Q)$ is f-CS in L as $g$is f- ℂℕ map. Since $h\circ g$ is fsgb-CM,$(h\circ g)\big(g^{-1}(Q)\big)=h(Q)$ is fsgb-CM in *N*. Hence $h:M\to N$ is fsgb-CM.

(ii) Let Q be f-CS in L. Then $(h\circ g)(Q)$ is fsgb-CS in N and hence $h^{-1}(h\circ g)(Q)=g(Q)$ is fsgb-CS in M. Since h is fsgb-irr and injective. Therefore g is fsgb-CM.

Theorem 6.10. If P is fsgb-CS in L and $g:L\to M$ is bijective fuzzy- ℂℕ map and fsgb-CM, then $g(P)$ is fsgb-CS in *M*.

Proof. Let $g(P)\le Q$ where Q is f-OS in M .Since g is fuzzy- ℂℕ map, $g^{-1}(Q)$is f-OS containing P. Therefore $\text{bCl}(P)\le g^{-1}(Q)$ as P is fsgb-CS. since g is fsgb-CM, $g(\text{bCl}(P))$ is fsgb-CS containing in the f-OS in Q, which implies $\text{bCl}(g(\text{bCl}(P)))\le Q$ and hence$\text{bCl}(P)(g(P)\le Q$. So $g(P)$is fsgb-CS in *M*.

Theorem 6.11. If $g:L\to M$ is fsgb-CM and $h:M\to N$ is fsgb*-CM then $h\circ g$ is fsgb*-CM.

Proof. For a fuzzy closed set in L, $g(P)$ is fsgb-CS in M . Since$h:M\to N$ is fsgb*-CM and $h(g(P))$ is fsgb-CS in *N*. $h(g(P))$= $(h\circ g)$ (P) is fsgb-CS in *N*. Hence $h\circ g$ is fsgb*-CM..

Theorem 6.12. Let $g:L\to M,h:M\to N$ be two maps such that $h\circ g:L\to N$ is fsgb*-CM.

(i) If g is fsgb-ℂℕ map and surjective, then h is fsgb-CM.

(ii) If h is fsgb-irr and injective, then g is fsgb*-CM.

Proof.

(i) Let P be fuzzy-CS of M. Then $g^{-1}(P)$ is fsgb-CS in L as g is fsgb- ℂℕ. Since $h\circ g$ is fsgb*-CM, $(h\circ g)$ $(g^{-1}(P))=$ h(P) is fsgb-CS in *N*. Hence $h:M\to N$ is fsgb-CM.

(ii) Let P be a fsgb-CS in L. Then $(h\circ g)$ (P) is fsgb-CS in *N*. Since h is fsgb-irr and injective $h^{-1}(h\circ g)(P)=g(P)$ is fsgb-CS in *M*. Hence g is fsgb*-CM.

Theorem 6.13. Every fb-closed (fgb-closed, fbg-closed) map is fsgb-closed.

Proof. Consider $g:(L,\tau)\to(M,\sigma)$ be a fb-closed (fgb-closed, fbg-closed) map. Consider Q be f-CS in L. Then $g(Q)$ is a f-CS in M, as g is a fb-closed (fgb-closed and fbg-closed) map. Thus $g(Q)$ is a fsgb-CS in M, as every fb-closed (fgb-closed and fbg-closed) set is a fsgb-CS. Therefore g is fsgb-CM.

The below illustration shows that the inverse implication of above theorem is untrue.

Example 6.14. Consider $L=\{d,e,f\}$, $M=\{d,e,f\}$

Let the f-sets be

$$P=\{(d,0.5),(e,0.2),(f,0.6)\}$$

$Q=\{(d,0.7),(e,0),(c,0.4)\}$ and $R=\{(d,0.5),(e,0.8),(c,0.4)\}$

Let $\tau = \{0, P, 1\}$ and $\sigma = \{0, Q, 1\}$. Define $g: (L, \tau) \rightarrow (M, \sigma)$ by $g(d) = g(e) = d$ and $g(\mathrm{f}) = \mathrm{f}$. Then the only f-CS in L is R and $g(R)$ is not a fb-CS and fgb-CS in M. Therefore g is a fsgb-CM but not a fbg-CM and a fbg-CM.

7. Conclusions:

It is curious to work on the compositions of weaker and stronger forms of mappings, as well as other properties of fsgb-closed set. Other forms of generalized closed fuzzy sets can be tried with mapping compositions.

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