

A Structure for the Most Effective Investment Strategy in a Pandemic Regional Sectors

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Abstract: That indeed happened that the COVID-19 pandemic caused the most serious economic disruptions with gross impacts on income, business, and quality of life as could be measured by declines in GDP across regions, countries, and continents. Recovery from pre-pandemic economic conditions will demand strategic intervention by both economic and political decision-makers. This paper presents a mathematical model for optimizing investment strategies in an environment of productivity fluctuations over time. Their expressed objectives in this model include achieving maximum revenue generation under the constraints imposed by the pandemic and thus cushioning the economy against possible damage resulting from lockdown measures. A strong point of this model lies in its cookie-cutter approach-spanning different economic units: regions, industries, enterprises, investment sectors, etc. With it also, policymakers can avail two options for strategies to invest in maximizing state revenues or reducing economic disparity associated with region differences per se. In contrast to previous models, this one generalizes other parameters that are not limited to lockdown due to a pandemic but can also include changes related to seasons. With those changing parameters, this model surely makes an investment decision-making system robust during periods of economic uncertainty.

Keywords: Optimal Control Theory, Revenue Maximization, Strategic Investment Planning, Economic Disparity Reduction, Pandemic-Induced Economic Adjustments, Regional Growth Optimization, Mathematical Modeling in Economics.

1. INTRODUCTION

It was the COVID-19 pandemic that brought a larger blow on the worldwide economy, leading sectors to drastic disruptions. As industries experience changes in emerging

unemployment and GDPs declining, alongside shifts in market demand, production halts are now common occurrences worldwide in the economies. Thus, governments and policy-makers are tasked with coming up with new ways to deal with these economic meltdowns by pursuing long-term stability and growth. One of the paramount concerns in the aftermath of the pandemic is how to optimize investment strategy given the context of regional lockdowns and slowdowns in the economy. Lockdown measures are necessary for controlling the spread of the virus, but they aggravate the financial uncertainties experienced by most vulnerable sectors and small businesses. This raises a critical urgency for models that are able to assist decision-makers for allocation of resources; while also ensuring economic recovery, it minimizes the gap on regional disparities. In this connection, a mathematical model is proposed to optimize investment decision under fluctuating productivity conditions. The model then serves as a way to maximize revenues subject to the restrictions imposed by lockdowns. This method's key strength is its flexibility: applicable across any economic unit, be it at the level of regions, industries, or individual businesses. It equally affords decision-makers two different strategic routes: that of increased state revenues through investments in high yields on one hand, and the other, which aims at reducing inequalities in the economy through balanced regional development.

The COVID-19 pandemic has caused unprecedented economic disruptions, leading to significant shifts in investment strategies. Governments worldwide implemented regional lockdowns to curb the spread of the virus, which directly impacted financial markets, consumer behavior, and business operations [1]. As economic activity declined, investors faced heightened uncertainty, requiring adaptive strategies to mitigate risks and capitalize on emerging opportunities [2]. Historical data from past pandemics, such as the Spanish Flu, provides insights into the long-term economic effects and potential recovery patterns [3]. Investment decisions during a crisis are influenced by financial fragility, volatility, and economic policies implemented to stabilize markets [4]. Studies suggest that pandemic-induced uncertainties alter corporate financial strategies, affecting stock prices, capital allocation, and economic growth expectations [5, 6]. Furthermore, research highlights the importance of firm resilience, as companies with strong financial structures tend to perform better during crises [7]. The adoption of alternative financial models and predictive analytics has played a crucial role in mitigating investment risks during volatile periods [8, 9]. Additionally, scholars emphasize the need for adaptive financial planning, integrating risk management strategies to address future economic shocks [10, 11]. By analyzing pandemic-induced market shifts, investors can optimize their strategies to navigate uncertainties and improve long-term returns [12, 15].

2. Model Description and Analysis:

This model builds upon the work of Pražák (see [16]), which is focused on a two region case and provides certain conditions for optimality. For this research, we made a number of enhancements to the out of the box model. First, we changed the model so it has N regions and also added an explicitly time varying function that accounts for lockdowns or seasonal changes in productivity. A second and significant change is the selection of a particular class of control that allows the use of different investment approaches. The primary focus is to enable the selection of strategies that will help in minimizing inequalities among the regions wherever required. In addition, we added a feature that guarantees that a certain proportion of the income of each region is invested back in the region. Our solution is novel and in sharp contrast to the

approach described in [17]. Calculating the optimal strategy is quite difficult, so we tried to analyze it through numerical means.

I. Model Description

In this section, we consider the following optimization problem: Given a final time T , the objective is to determine $(u_i(t))_{i=1}^N$ that maximize:

$$Y(T) = \sum_{i=1}^N Y_i(T)$$

Subject to the following constraints:

$$\frac{dk_i(t)}{dt} = \sum_{j=1}^N b_j k_j(t) c_j(t) + s_i b_i k_i(t) c_i(t) = v_i(t)$$

And

$$\sum_{j=1}^N v_j(t) = 1, u_i(t) \in [0, 1], t \in [0, T]$$

This is an optimal control problem aimed at maximizing the total income $Y(T)$ by choosing the optimal investment strategy. The function $k_i(t)$ represents the capital stock of region i at time t , and its evolution is governed by the first constraint. This constraint models the accumulation of capital in each region, with the regional output-capital ratio b_i and the regional saving ratio s_i both being positive.

Furthermore, a portion $s_i > 0$ of the output-capital ratio in region i is reinvested into the same region, while the remainder can be allocated to other regions. The control variables $u_i(t)$ indicate the fraction of total investment allocated to region i , as dictated by the second constraint. The parameter α specifies the minimum share of the total investment that must be allocated to each region, reflecting the desired policy goals. For instance, setting $\alpha = 1$ would mean equal distribution of investment across all regions, which helps reduce disparities. Alternatively, if $\alpha = 0$, there is no minimum allocation required for each region, which could potentially exacerbate inequalities.

Finally, the function $c_i(t)$ is assumed to be piecewise continuous and models the effects of factors like lockdowns or other disruptions that can alter the productivity of each region.

II. Necessary Conditions for the Optimal Solution

Investments need to be allocated appropriately and for that, we make use of the Pontryagin Maximum Principle. Now the first step is to look at the Hamiltonian that goes with the optimization Problem (P) .

Let the vectors $K = (k_1, k_2, \dots, k_N)$, $U = (u_1, u_2, \dots, u_N)$, $V = (v_1, v_2, \dots, v_N)$, and $P = (p_1, p_2, \dots, p_N)$ represent the capital, investment strategy, auxiliary variables, and adjoint variables, respectively. The Hamiltonian takes the following form:

$$H(K, V, P) = \sum_{i=1}^N p_i(t) v_i(t) + \sum_{j=1}^N s_j b_j k_j(t) c_j(t) + \tilde{s}_i b_i k_i(t) c_i(t)$$

Where p_i , for $i = 1, 2, \dots, N$ are the adjoint functions corresponding to the optimal control problem. The adjoint equations are given by:

$$\frac{dp_i(t)}{dt} = H_{k_r} = \sum_{j=1}^N p_i(t)v_j(t)s_r b_r c_r(t) + p_r(t)\tilde{s}_r b_r c_r(t)$$

With the terminal condition:

$$p_r(T) = b_r c_r(T)$$

For $r = 1, 2, \dots, N$. It is easy to see that each adjoint function p_i is decreasing since its derivative is negative, and the adjoint variables p_i are positive.

Using the Pontryagin Maximum Principle, we derive the optimal state $\hat{k}$ and optimal control $\hat{U}$. The condition becomes:

$$H(\hat{k}, \hat{V}, P) \geq H(\hat{k}, V, P)$$

Which leads to the following inequality:

$$\begin{aligned} \sum_{i=1}^N p_i(t)v_i(t) + \sum_{j=1}^N s_j b_j \hat{k}_j(t)c_j(t) + \tilde{s}_i b_i \hat{k}_i(t)c_i(t) \\ \leq \sum_{i=1}^N p_i(t)\hat{v}_i(t) + \sum_{j=1}^N s_j b_j \hat{k}_j(t)c_j(t) + \tilde{s}_i b_i \hat{k}_i(t)c_i(t) \end{aligned}$$

From basic calculus, we know that:

$$\left(\sum_{j=1}^N s_j b_j \hat{k}_j(t)c_j(t) \right) \left(\sum_{i=1}^N p_i(t)(v_i(t) - \hat{v}_i(t)) \right) \leq 0$$

Since the first sum is always positive, we conclude that:

$$\sum_{i=1}^N p_i(t)(v_i(t) - \hat{v}_i(t)) \leq 0$$

Next, recalling that:

$$v_l(t) = 1 - \sum_{r=1, r \neq l}^N v_r(t)$$

And

$$\hat{v}_l(t) = 1 - \sum_{r=1, r \neq l}^N \hat{v}_r(t)$$

We can substitute this into the previous inequality to obtain:

$$\sum_{i=1, i \neq l}^N (v_i(t) - \hat{v}_i(t))(p_i(t) - p_l(t)) \leq 0 \Leftrightarrow \sum_{i=1, i \neq l}^N (u_i(t) - \hat{u}_i(t))(p_i(t) - p_l(t)) \leq 0$$

This offers the prerequisite for the ideal outcome we are looking for. Let's pick an arbitrary moment t and $l^*(t)$, the region where $p_{l^*}(t)$ is at its biggest, in order to talk about the best course of action. It is crucial to remember that the moment t determines $l^*(t)$.

This implies that:

$$p_i(t) \geq p_{l^*}(t) \text{ for all } i = 1, 2, \dots, N$$

Thus, the necessary condition holds with $l = l^*$ only if:

$$\hat{u}_{l^*} = 1 \text{ and } \hat{u}_i = 0, \text{ for } i = 1, 2, \dots, N, i \neq l^*$$

The model's intricacy made it challenging to determine the essential conditions of optimality using direct calculus. As a result, we'll employ the numerical solution shown in the next section.

3. Numerical

With the help of the control function u_i optimality condition and the knowledge that $v_i(t) = \frac{\alpha}{N} + u_i(t)$, we can rebuild the head joint system as follows:

$$\dot{p}_r(t) = - \sum_{i=1}^N p_i(t) v_j(t) s_r b_r c_r(t) - \tilde{s}_r b_r p_r(t) c_r(t)$$

Where:

$$l^*(t) = \arg \max_{l \in \{1, \dots, N\}} p_l(t)$$

The final function of the head joint is known (it is independent of $k_i(T)$). Runge-Kutta scheme in Python has been implemented using the hierarchical Numpy, which can be used to numerically solve the aforementioned system of differential equations. By doing this, wearable technology can determine which time period we will inject the cash into and calculate $l^*(t)$ (discretized) time. Stated differently, this offers the most effective funding plan.

Case Studies for Lockdown Scenarios

First Example: Gradual Regional Impact

We consider a scenario with $N = 5$ regions, an initial capital of $k_i(0) = 1$ for all regions, and the following parameters:

$$b = [0.12, 0.118, 0.115, 0.12, 0.117]$$

$$s = [0.2, 0.21, 0.195, 0.2, 0.198]$$

$$\tilde{s} = [0.11, 0.12, 0.13, 0.11, 0.1]$$

The productivity function $c_i(t)$ is defined as:

$$c_i(t) = 1 - \frac{(t - (i - 1))(i + 1 - t)}{2}, \text{ if } t \in [i - 1, i + 1], \text{ otherwise } 1.$$

Similar to a phased lockout effect, this demonstrates a stepwise pattern where productivity declines and then progressively recovers over time for all regions. The chosen investing strategy looks very advanced because it generates a final total revenue of about $y(T) \approx 0.838$.

The total income of $y(T) \approx 0.838$ is the result of a straightforward investment heuristic that only distributes money to the region with the highest productivity function of $g_i(t) = s_i b_i c_i(t)$ at each time step. This validates simpler investment strategies over the one that is numerically optimized.

Second Example: Abrupt Lockdown Effects

A fresh collection of productivity functions is presented to simulate a situation in which the lockdown brings about more sudden effects, utilizing the identical regional parameters as in the initial instance:

$$\begin{aligned} c &= [1, 1, 1, 1, 1] \text{ for } t \in [0, 2], \\ &[0.8, 0.7, 0.5, 0.5, 0.6] \text{ for } t \in [2, 4], \\ &[0.6, 0.4, 0.8, 1, 0.7] \text{ for } t \in [4, 6], \\ &[1, 1, 0.7, 0.6, 0.8] \text{ for } t \in [6, 8], \\ &[1, 1, 1, 1, 1] \text{ for } t \in [8, 10], \end{aligned}$$

The initial scenario and the optimal investment approach vary greatly, emphasizing the importance of tailored strategies according to the economic conditions. Establishing $\alpha = 0.5$ further lessens disparities by allocating investments more uniformly.

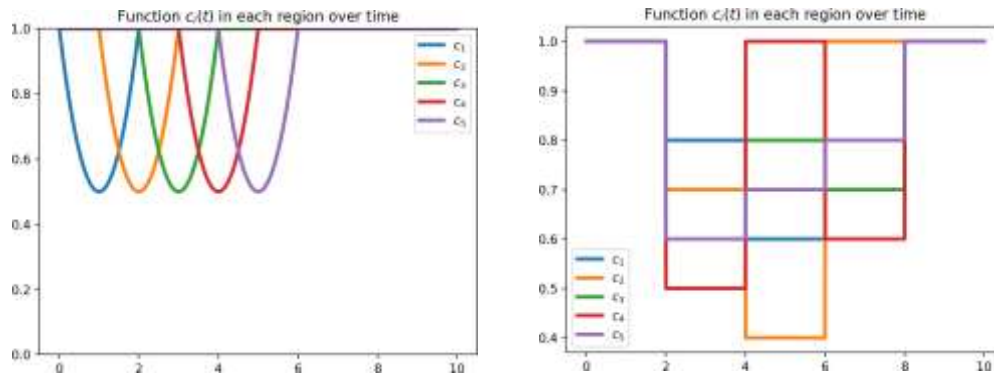


Figure 1. Shows the function $c_i(t)$ as a function of time for both the first and second examples (left and right, respectively).

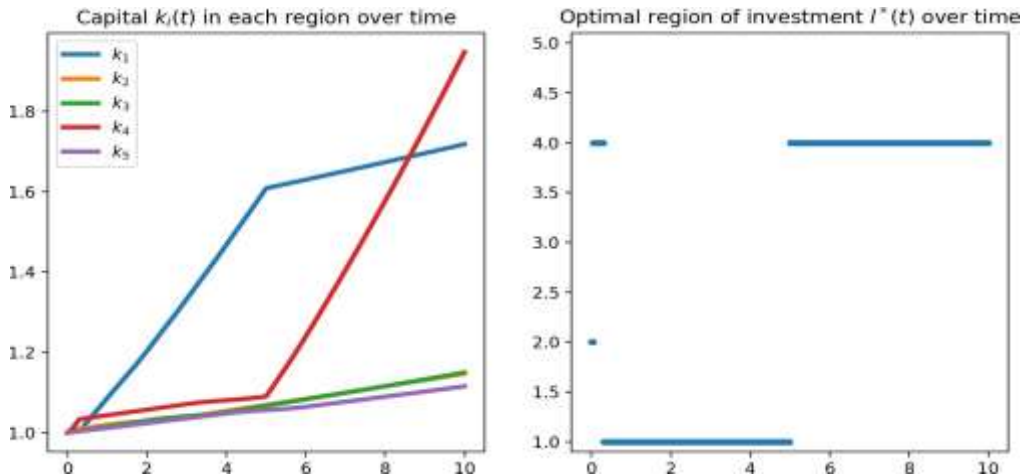


Figure 2. Capital $k_i(t)$ and the region's ideal index $l^*(t)$ over time, with $\alpha = 0$, are shown on the left and right, respectively.

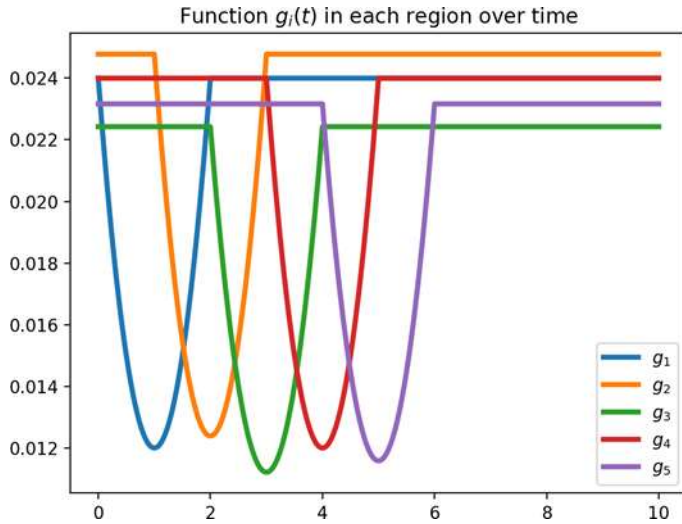


Figure 3. Representation of the function $g_i(t)$ versus time.

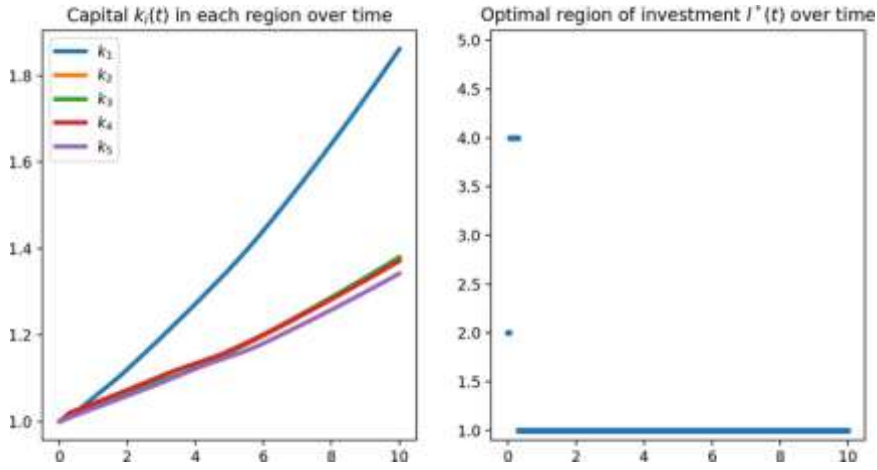


Figure 4. Using $\alpha = 0.8$, the evolution of the region's ideal index $l^*(t)$ and capital $k_i(t)$ over time are shown on the right and left, respectively.

In the second case, we have used $\alpha = 0$ to depict the results on Figure 5. It is evident once more that the plan is not clear. Moreover, given the same α , which explains the change in function $c_i(t)$, it is obviously not the same as in the first example.

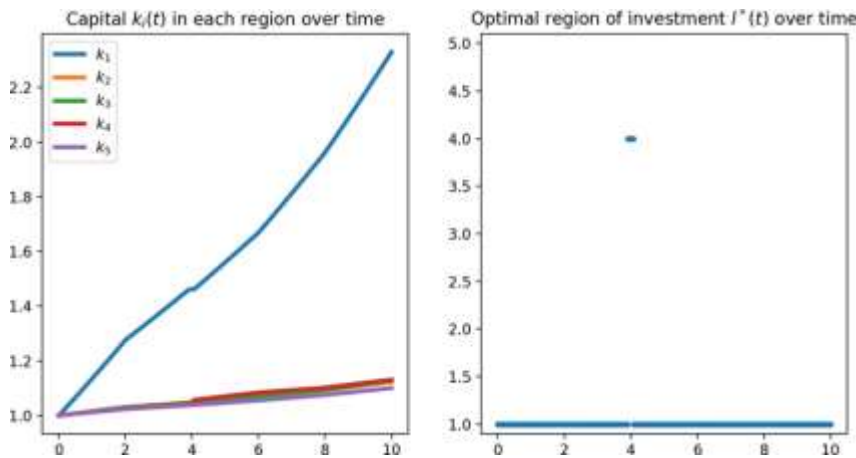


Figure 5. The capital $k_i(t)$ and the region's ideal index $l^*(t)$ are shown on the left and right, respectively, with $\alpha = 0$.

In Figure 6, we determined the optimal strategy with $\alpha = 0.5$ to emphasize the influence of the parameter α . Again, it is evident that the optimal course of action changes and that the inequalities lessen by increasing α .

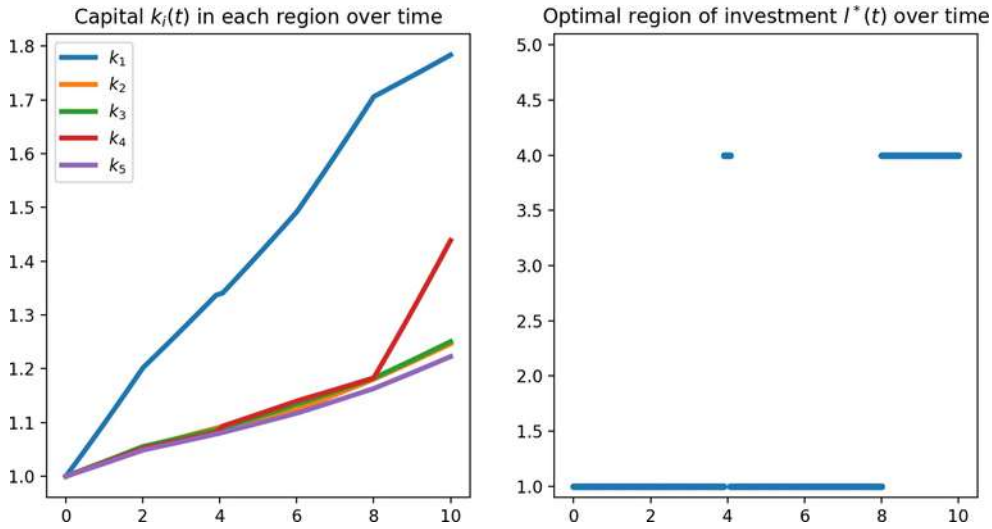


Figure 6. Changes in the region's ideal index $l^*(t)$ and capital $k_i(t)$ over time are shown on the right and left, respectively.

Second productivity Variations

In the last scenario, three regions with cyclically fluctuating economic activity are used to mimic a seasonal effect:

$$b = [0.2, 0.18, 0.2]$$

$$s = [0.2, 0.2, 0.2]$$

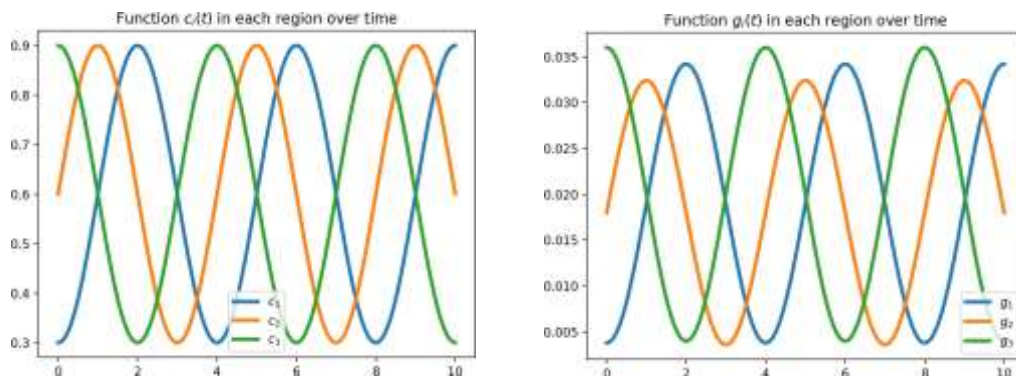
$$\tilde{s} = [0.1, 0.1, 0.1]$$

The productivity function follows a cosine-based periodic pattern:

$$c_i(t) = \cos\left(\frac{(t-i)\pi}{2}\right) \times 0.4 + 0.5$$

This would mimic seasonal changes of summer high for coastal and winter peak for alpine areas.

Remarkably, the best course of action recommends continuously investing in the first region, which results in a $y(T) \approx 0.453$ ultimate income. A naive method that follows the region with the highest productivity function $g_i(t)$ at each time step yields a lower total income $y(T) \approx$



0.392, which is counterintuitive. This emphasizes even more how crucial long-term, strategic investment is to maximizing profits in the short term.

Figure 7. The function $g_i(t)$ and the function $c_i(t)$ versus time are shown on the right and left, respectively.

The ideal solution of our optimization issue (assuming $\alpha = 0$) is depicted in Figure 8, and as we can see, the best course of action is to simply inject the money in the same zone (region 1). The ultimate value in this instance is $y(T) \approx 0.453$.

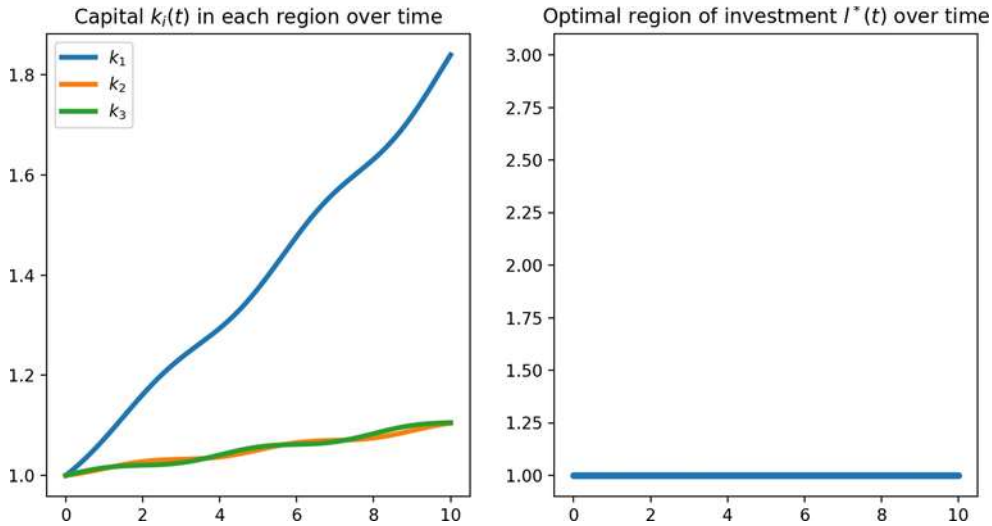


Figure 8. The ideal index $l^*(t)$ of the region and the capital $k_i(t)$ over time are shown on the right and left, respectively.

This result could seem contradictory, and one could argue that investing in the region that generates the most would be a more successful strategy. That is, to take:

$$l^*(t) = \arg \max_{l \in \{1, \dots, N\}} g_l(t)$$

We have illustrated the results obtained with this second strategy in Figure 9, to compare the two strategies against one another. Here, the ending value $y(T) \approx 0.392$ is lower than the value that the best possible method (as expected) would have generated.

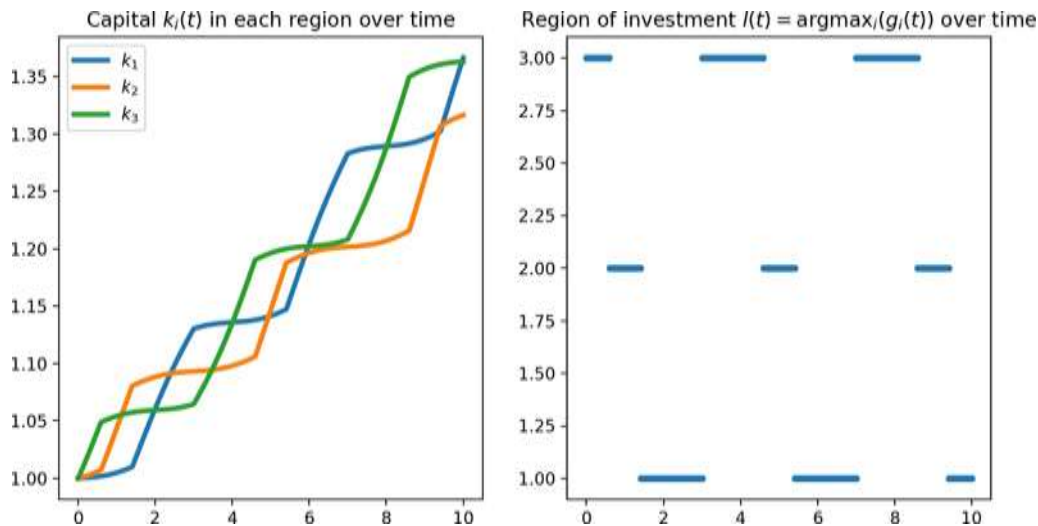


Figure 9 shows the change in the capital $k_i(t)$ on the left and the index $l^*(t)$ on the right over time.

4. Conclusion and Discussion

The economic impact of the COVID-19 pandemic has been profound, disrupting markets, industries, and regional economies worldwide. This study introduced a mathematical model designed to optimize investment strategies during such uncertain times, helping policymakers and investors allocate resources more effectively. The model accounts for regional disparities, fluctuating productivity levels, and the long-term effects of lockdowns, offering a strategic approach to revenue generation and economic stability.

One key takeaway is that investment strategies must be adaptable, considering both immediate economic shocks and long-term recovery prospects. Our findings suggest that blindly investing in the most productive regions at any given time may not always yield the best overall outcomes. Instead, a balanced and strategic investment approach—one that factors in fluctuations in regional productivity, seasonal economic shifts, and potential lockdown scenarios—proves to be more effective. The numerical simulations highlighted how different lockdown intensities impact optimal investment choices. In scenarios with gradual economic decline, investments can be strategically shifted to mitigate income disparities. However, in cases of abrupt lockdowns, the model recommends more evenly distributed investments to ensure financial stability across regions. Additionally, the model demonstrated that short-term profit-maximizing strategies do not always align with long-term economic resilience, reinforcing the need for thoughtful financial planning during crises. These insights emphasize the importance of flexible investment policies, particularly during periods of economic uncertainty. Policymakers can use such models to develop intervention strategies that support economic recovery while minimizing disparities between regions. Future research could further refine this model by incorporating real-world data on pandemic recovery trends, government stimulus effects, and evolving market behaviors. By integrating such improvements, investment strategies can become even more robust, ensuring that economies are better prepared for future crises.

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