

Basic operations with rational numbers and didactic mediation of the place value board in first grade students in Huánuco

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SUMMARY

The purpose of the scientific inquiry was to verify that the didactic mediation of the place value board improved the learning of basic operations with decimal rational numbers of students of the first grade of secondary school in Huánuco; for this purpose, an explanatory research and quasi-experimental design was formulated; the accessible population was 78 students and the sampling used was non-random. with 37 in the experimental group and 41 in the control group; Three written tests were used, with the name of initial test, process test and final test, with ten questions each test and two points per question, so the vigesimal scale was used: [00 - 20]; the data processing was carried out within the framework of descriptive statistics and inferential statistics, obtaining the following result: the calculated Z value was 3.76, which was located in the rejection zone with respect to the critical Z equal to 1.96, which is a critical value for 95% of reliability and 5% of significance; in this sense, the null hypothesis was objected to and the alternative hypothesis was admitted, because there were sufficient indications that proved that the learning of basic operations with decimal rational numbers improved with the didactic mediation of the place value board in students of the first grade of secondary school in Huánuco.

Keywords: Learning basic operations, addition, subtraction, multiplication, division, decimal rational number, place value board.

ABSTRACT

The object of the scientific investigation was to verify that the didactic mediation of the place value board improved the learning of basic operations with decimal rational numbers of the students of the first grade of secondary school in Huánuco; To this end, an explanatory research with a quasi-experimental design was formulated; The accessible population was 78 students and the sampling used was non-random, with 37 in the experimental group and 41 in the control group; Three written tests were used, named initial test, process test and final test, with ten questions each test and two points per question, so the vigesimal scale was used: [00 - 20]; The data processing was carried out within the framework of descriptive statistics and inferential statistics, obtaining the following result: the calculated Z value was 3.76, which was located in the rejection zone with respect to the critical z equal to 1.96. which is a critical value for

95% reliability and 5% significance; In this sense, the null hypothesis was objected, and the alternative hypothesis was admitted, because there was sufficient evidence to prove that the learning of basic operations with decimal rational numbers improved with the didactic mediation of the place value board in the undergraduate students. first year of secondary school in Huánuco.

Keywords: Learning basic operations, addition, subtraction, multiplication, division, decimal rational number, place value board

INTRODUCTION

It was observed that students from secondary school institutions in the district of Huánuco showed difficulties in writing and reading rational numbers in their decimal form, a fact that had an impact on the generation of learning with confidence and expertise in basic operations with these numbers (Blandón et al., 2022).

Among the difficulties that the students externalized was the positional location of each digit before and after the decimal point because in each basic operation at least two rational numbers interact and the result is a third number of the same kind (Medina, 2016).

The other two difficulties consisted of writing and reading the rational numbers that they manipulated in each of the operations they were asked to perform (Pineda, 2017).

The universalized numbering system is the *decimal* because ten units of one order form one unit of the next order; In addition, the value of each figure depends on its position in the number and this evidences its characteristic of *positional* (Carrasco, 2020).

Rational numbers have two parts, to the left of the decimal point is the whole part and to the right the decimal part (Albornoz et al., 2014), as can be seen in the following table:

Table 1. Place value board of the base 10 numbering system

Tens of a million	Million Unit	Hundred thousand	Ten thousand	Mil Unit	Hundred	Ten	Unit	Decimal Point	Tenth	Hundredth	Thousandth
Dmi	UMi	CM	DM	UM	C	D	Or	,	d	c	m
8	2	3	9	3	1	4	6	,	2	6	4

It is suggested to write in the way presented in table 1 and the reading of this would be: *eighty-two million three hundred ninety-three thousand one hundred forty-six units, comma, two hundred sixty-four thousandths*. Writing and reading is the recognition of the digits from 0 to 9 and thus apprehend the place value of each digit and apply them in mathematical operations understanding their applicability in the problems of the environment (Coy & Nieto, 2020).

Writing involves doing it first the whole part and then the decimal part with the separation of a decimal point of a given number, this implies the visualization of the place value of each figure and its value depends on the place of location on the positional board of the number being analyzed; In addition, it must be assumed that there are four skills directly linked to operations with the place value numbering system, and they are: counting, splitting, grouping and relating numbers, in that sense, it is assumed that the skills are developed in parallel with the understanding of the system (Zúñiga, 2012).

The correct way to write and apprehend a decimal number allows comparison to establish the relations of inequality, in addition, its location on the number line, the approximations or roundings to any position, it also allows to establish the link of the decimal fraction with a decimal number and to carry out the basic operations with the

rational numbers complying with the rules established in each of them (Nardoni, 2014) and (Caviedes et al., 2023).

In the context described, the use of teaching materials during the execution of mathematics learning sessions is essential, which justifies the use of the place value board when developing and creating learning activities on basic operations with rational numbers in decimal mode, whose application in the classroom must be interactive (Navarrete, 2017).

The application of the place value board implies promoting didactic games during the learning of mathematics because they help to achieve the goals set and put as many senses as possible in interaction, so the learning of basic operations with rational numbers would be quite beneficial (Ricca & Ricca, 2021).

The object of the inquiry was to check that the didactic mediation of the place value board allows improving the learning of basic operations with rational numbers in the units of analysis, because they do so through the interaction of hearing, sight and touch, allowing their cognitive development with greater advantage than those who develop the mathematics course in a behaviorist way (Rodríguez, 2019).

The importance of the level of learning addition and subtraction in Q with the didactic mediation of the place value board is that this benefit is determined through an inquiry and as such can be applied in any other scenario, with the only recommendation being to contextualize the data collection instrument to the new scenario (Gallego & Uzuriaga, 2015).

Emphasize in classes, that the first thing to read is the whole part followed by the word units and the decimal comma is put and then the decimal part is read naming the place occupied by the last digit, in that sense, if the decimal has zero units, only the decimal part is read naming the place occupied by the last digit, for example, the following rational number: 56005942,0010002 which would read: *fifty-six million five thousand nine hundred and forty-two unitscoma one thousand two ten millionths* (Diaz & Silva, 2021).

In both the addition and subtraction of rational numbers, it is elementary that the decimal point is aligned one below the other between the addends, and between the minuend and subtraction, respectively, as can be seen in Table 2, so it is necessary to abound in examples proposed by the teacher and the same units of analysis in sufficient quantity. Until the use becomes familiar to them, this will allow students to propose their innovations in the operational process of addition, subtraction, multiplication and division, in this sense, one of the first innovations of the units of analysis would be that they no longer always start on the right but on the left. using place values; or perhaps, they would do it by grouping them together to convert the addends to easily manageable numbers by their place value (García, 2015).

In table 2, the summands are located according to the rules, also the sum by columns, in addition, the tens that are taken and finally the sum, which by the way reads as follows: *twenty-one million eighty-three thousand three hundred seventy-five, point eight hundred and forty-two thousandths*.

The units of analysis understand and apprehend the didactic mediation of the place value board from primary school and consolidate it in the first grades of secondary school, otherwise the consequences will be notorious: they will not know how to read or write a rational number, nor will they be able to write a dictated rational number, which will make them dependent on the use of technology (Villalba, 2023).

The apprehension of the place value board is a constructive learning through the process of assimilation and accommodation of the student by relating and fitting all the characteristics of the aforementioned didactic mediator within the structures of his knowledge and the ability to incorporate it depends on the level of cognitive development of the units of analysis (Napán, 2018).

Table 2. Example of didactic mediation of the *place value board* in addition

	Tens of a million	Million Unit	Hundred thousand	Ten thousand	Thousand Unit	Hundred	Ten	Unit	Coma	tenth	hundredth	thousandth
Cmi	Dmi	UMi	CM	DM	UM	C	D	Or	,	d	c	m
Adding 1					2	6	9	8	,	0	0	7
Adding 2							6	4	,	8	9	
Adding 3		8	2	8	0	7	0	1	,	0	8	
Adding 4	1	2	7	9	9	9	1	1	,	8	6	5
Summing by columns	1	10	9	17	11	22	16	14	,	16	23	12
Taken	+1	+1	+1	+1	+2	+1	+1	+1		+2	+1	
SUM	2	1	0	8	3	3	7	5	,	8	4	2

In the multiplication of decimal rational numbers, the factor numbers are placed vertically and multiplied as if they were natural numbers, then, the decimal point in the product is placed counting from right to left according to the number of decimals the factors have.

The above can be seen in table 3, where factors 1 and 2 have been placed vertically, in addition, the partial products 1, 2 and 3 are natural numbers and in the product 1a as a decimal has been placed in place three counted from right to left because factor 1 has two decimal places and factor 2 has one decimal. The result obtained is written and read: *sixty-four thousand six hundred and thirty-four, point five hundred and forty-four thousandths*.

Table 3. Example of didactic mediation of the *place value board* in multiplication

	Tens of a million	Million Unit	Hundred thousand	Ten thousand	Thousand Unit	Hundred	Ten	Unit	Coma	tenth	hundredth	thousandth
	Dmi	UMi	CM	DM	UM	C	D	Or	,	d	c	m
Factor 1						7	0	4	,	0	8	
Factor 2							9	1	,	8		
Partial Product 1						5	6	3	2	6	4	
Partial Product 2						7	0	4	0	8		
Partial Product 3				6	3	3	6	7	2			
PRODUCT				6	4	6	3	4	,	5	4	4

In the division of decimal rational numbers, both the dividend and the divisor must be observed and determine which of them has the highest number of decimals, if one of them has one and the other three, then both are multiplied by a thousand and divided as whole numbers.

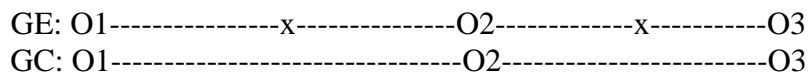
Table 4. Example of didactic mediation of the *place value board* in division

	Tens of a million	Million Unit	Hundred thousand	Ten thousand	Thousand Unit	Hundred	Ten	Unit	Coma	tenth	hundredth	thousandth
	Dmi	UMi	CM	DM	UM	C	D	Or	,	d	c	m
Dividend						1	0	4	,	0	2	
Divisor								0	,	0	0	4
Dividend per 1000			1	0	4	0	2	0				
Divider by 1000								4				
QUOTIENT				2	6	0	0	5				

The quotient is written and read as follows: *twenty-six thousand five units*. The rules applied with the mediation of the place value board are in the mathematical literature that were lost until the sixties and are accentuated in the seventies with the appearance of calculators and other digital tools.

MATERIALS AND METHODS

The scientific research carried out was explanatory (Paragua et al., 2022), the design used was quasi-experimental (Norberto et al., 2018), the scheme of which is as follows:



The diagram shows three observations to both groups with three different tests: the initial diagnostic test of previous knowledge, the process and final with didactic mediation of the place value board in the experimental group, while the control group fulfills its role as controller.

The accessible sample population consisted of seventy-eight, with GE=37 and GC=41, students of the first grade of secondary school enrolled in the different educational institutions of the district of Huánuco, enrollment 2023; The sampling applied was intentional.

Each test with ten development questions valued at two points per question and was graded with the vigesimal scale. The data were processed and the central tendency, dispersion and shape statistics were found to be able to interpret the behavior of the experimental group with respect to the level of learning of basic operations with the decimal rational numbers with the didactic mediation of the place value board; Then, a hypothesis test of the mean difference was done, with the distribution Normal z (Paragua et al., 2018).

RESULTS

The results of the fieldwork, both of the EG and GC are shown in the following table:

Table 5: Descriptive analysis of the level of prior knowledge, learning in process and final learning on basic operations with decimal rational numbers in students of the first grade of secondary school in Huánuco, enrollment 2023. GE and GC.

Statisticians	1st- GE-PI	1st- GE-PP	1st- GE-PF	1st- GC-PI	1st- GC-PP	1st- GC-PF
Stocking	8,22	10,19	12,30	8,39	10,17	10,98
Median	8,00	10,00	12,00	8,00	10,00	11,00
Fashion	8,00	9,00	13,00	8,00	9,00	12,00
Standard deviation	1,72	1,76	1,53	1,84	1,80	1,57
Sample variance	2,95	3,10	2,33	3,39	3,25	2,47
Asymmetry coefficient	0,24	0,89	0,01	0,42	0,78	-0,77
Rank	7,00	7,00	7,00	8,00	7,00	7,00
Minimal	5,00	8,00	9,00	5,00	8,00	7,00
Maximum	12,00	15,00	16,00	13,00	15,00	14,00
n	37	37	37	41	41	41

Source: Initial Test (IP), Process Test (PP) and Final Test (PF).

The statisticians in the first three columns correspond to the EG, of which the first column describes the level of prior knowledge and indicates approximately forty-two percent of prior knowledge, an insufficient amount to generate successful learning on the problem topic, so they were fed back directly to the poorly solved test indicators. *Media* = 8,22

The dispersion of the level of learning, with the is low within the scale, in addition, configures a positive asymmetry and that means that the majority of the thirty-seven units of analysis have levels of previous knowledge below the Madia with a tendency towards the statistical. $Desviación\ estándar = 1,72$ $Coeficiente\ de\ asimetría = 0,24$ $Mínimo = 5$

The statistics of the PP and PF column of the EG are comparable because they are produced with the mediation of the place value board, in this context, the learning of the basic operations of decimal rational numbers with the mediation of the proposed didactic tool has improved from the feedback of previous knowledge, This is confirmed by the growth of measures of central tendency with a final mean equal to *twelve integers and thirty hundredths*.

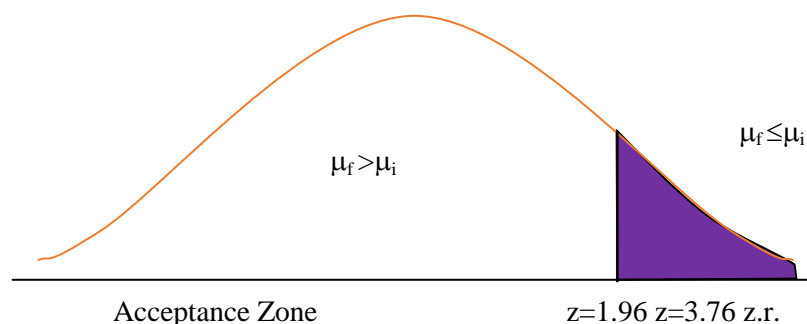
In addition, the dispersion measures represented by the standard deviation of *an integer*, with *seventy-six hundredths* decrease to *an integer*, with *fifty-three hundredths*, in that sense, the decrease means that the level of learning of basic operations with rational numbers was homogenizing.

The statistics in the fourth column correspond to the previous knowledge of the CG and are similar to those of the GE analyzed because they belong to the same accessible population under study, however, they were not programmed any feedback, nor did they benefit from the didactic mediation of the place value board, but the same evaluations were applied to them as to those of the EG because they were fulfilling their role as controllers.

The problem topic under study is also part of the curriculum development of the CG in charge of another teacher and the evolution of their learning that is observed through the statistics of the fifth and sixth columns show improvements with less intensity than those of the EG, likewise, the homogenization of learning is occurring with greater intensity than in the EG. The event described occurs in the interval [7 - 14] and there the statistic in a way like the: configures a negative asymmetry, indicating that most of them tended towards the statistical. $Coeficiente\ de\ asimetría = -0,77$ $Máximo = 14$

The hypothesis testing implied its formulation: and ; where, the alternate hypothesis shows that the test is one-sided right-tailed, because it is about verifying only one probability; In addition, the sample distribution appropriate to the scientific inquiry is the mean difference and the normal distribution was used $H_0: \mu_f \leq \mu_i$ $H_A: \mu_f > \mu_i$ Z , being calculated with the formula: , then: , then the value of the test Z is: $Z = \frac{\bar{\mu}_e - \bar{\mu}_c}{\sqrt{\frac{\delta_e^2}{n_e} + \frac{\delta_c^2}{n_c}}} Z = \frac{12,30 - 10,98}{\sqrt{\frac{2,33^2}{37} + \frac{2,47^2}{41}}} Z = 3,76$.

Graph 1: Hypothesis testing. Rejection and acceptance zone



The value $Z = 3.76$ is located to the right of $z = 1.96$; critical value for 95% reliability, that is, the null hypothesis is objected to and the alternative hypothesis is admitted, because there are sufficient indications that prove that the learning of basic

operations with decimal-rational numbers improves with the didactic mediation of the place value board in students of the first grade of secondary school in Huánuco.

DISCUSSION OF RESULTS

The level of prior knowledge of the units of analysis diagnosed at the beginning of the inquiry was insufficient for successful learning, which was probably mitigated with feedback sessions on prerequisite topics such as place value, decimal numbering and basic operations with them (Castorina, 1994) and (Salazar, 2018).

Learning about specific topics is of immediate achievement and constitutes the basis for the development of mathematical competencies, in this sense, the didactic mediation of the place value board allows the development of mathematical skills, such as: counting one by one from a quantity, counting by ten, from events in the environment, like: Juanito enters a bakery and sees twelve French-type breads, thirty wholemeal breads and twenty cariocas; So, one of many questions would be: How many loaves of bread did you see regardless of the type? (Cabanillas et al., 2021).

The question proposed to the units of analysis of the first grade allows us to determine the level of previous knowledge about the problem under study from real situations, a fact that requires a lot of practice and involvement on items such as: rules and properties of basic operations, writing and reading decimal numbers on the place value board (Montalvo et al., 2021).

The study aims to achieve the learning of basic operations with decimal rationals, promoting a process of enculturation, sharing and understanding the mathematical meaning through communication between the units of analysis, due to the fact that learning mathematics is a complex foundation that involves participating, socializing and interacting, which lead to the development of mathematical competencies mediated by the use of a didactic tool based on Events edited from the environment (López & Del Valle, 2017).

The development of basic operational skills is formed from the understanding of the partitioning of objects, in that sense, the analysis situation would be: a mathematics teacher brings to his class of twenty-eight students a bag with ninety-one candies of a soft nature. The teacher asks for an individual report on: how many tens are there?, how many dozens are there?, how many whole candies each one gets?, how many candies exactly do each one eat?, there can be many other questions (Rodríguez et al., 2022) and (Valbuena et al., 2021).

The grouping events reinforce and evaluate that the partitions of ten allow them to count and facilitate the basic operations, in addition to reading and writing them with increasing ease and speed, which is why the act described allows the student to form a particular and own numerical thinking from contextualized examples, because their formation and evolution is gradual as a direct function of the opportunities they have to think about numbers and use them in contexts of their environment (Parreño, 2017).

The units of analysis when using the place value board to execute the basic operations with the decimal-rational numbers must start by exercising the mastery of counting and grouping, it is these actions that allow them to understand and apprehend the decimal numbering system and complement it with reading and writing from interactive dictation; In addition, the skill of counting develops the ability to order and compare numbers and for this they must count in an ascending and descending way, count in jumps of two, three and more (Zúñiga, 2012).

The decimal numbering system is based on successive grouping, where units are grouped into tens; In addition, collections of ten tens are called hundreds; then, ten of these groupings are called thousands and so on, this form of grouping is recognized as a base ten system (Avalos, 2016) and (Benites et al., 2021).

One of the ways to confirm the apprehension of the use of the place value board is that, of the following group of numbers: 578; 704; 477; 7005.01007 The units

of analysis must read the given numbers, in addition, they must write the place value that the digit seven has, they must write the developed form of the given numbers, in addition, other activities that involve interaction between classmates (MINEDU, 2022).

The proposal to generate a better level of learning of basic operations with decimal-rational numbers with the didactic mediation of the place value board, according to the results obtained, show evidence of its effectiveness; that is, in addition to making it easier for them to master the operating rules of each basic operation, the units of analysis show improvements in terms of reading and writing decimal numbers, specifying the position of the units of thousands, hundreds, tens and units that support each of the quantities (Zorrilla et al., 2022).

The final results obtained show that the level of learning of basic operations with rational decimal numbers, with didactic mediation of the place value board, improves in students of the first grade of secondary school in Huánuco.

CONCLUSIONS

- The level of prior knowledge possessed by the units of analysis on basic operations with rational decimal numbers was approximately forty-two percent, which was insufficient to develop and achieve adequate learning of the problem subject under study.
- The level of learning of basic operations with decimal rational numbers improves during the didactic mediation of the place value board, confirmed by that of the experimental group, and at the same time begins to homogenize indicated by its asymmetry coefficient that is trending downwards. $Media = 10,19$
- The level of learning of basic operations with decimal rational numbers improves at the end of the didactic mediation of the place value board, confirmed by that of the GE, in turn, the level of learning acquired is homogenized indicated by its $Media = 12,30$ $Coeficiente\ de\ asimetría = 0,01$
- The horizontal comparison shows that the learning level of the EG goes to , that is, there is an improvement of 4.08 points on average; which indicates the effectiveness of the mediation of the place value board because it allows improving the learning of basic operations with rational decimal numbers. $Media = 8,22$ $Media = 12,30$
- The cross-comparison shows that the learning level of the EG goes to , that is, there is an improvement of 4.08 points on average; which indicates the effectiveness of the mediation of the place value board because it allows improving the learning of basic operations with rational decimal numbers. $Media = 8,22$ $Media = 12,30$
- The contrast of the general objective tells us that the value of Z equal to 3.76 is located to the right of critical z equal to 1.96 for 95% of reliability, that is, in the rejection zone, so the null hypothesis is objected to and the alternative hypothesis is admitted, because there are sufficient indications that prove that the learning of basic operations with decimal rational numbers improves with didactic mediation of the place value board in students of the first grade of secondary school in Huánuco.

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